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Neutrino Masses, Nuclear Matrix Elements, and the $0\nu\beta\beta$ Decay of ^{76}Ge*

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Abstract—Neutrino data, obtained from SNO, SK, CHOOZ, KamLAND, and WMAP, are used to establish the upper limit of $\langle m_\nu \rangle$ relevant for the $0\nu\beta\beta$. The decay of ^{76}Ge is discussed within different light-neutrino mass spectra and with different nuclear matrix elements. © *2004 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

Our present understanding of the properties of the neutrino has been dramatically advanced by the results of various experiments [1–5]. The experimental evidence has confirmed the existence of neutrino flavor oscillations [6]. In addition to neutrino flavor oscillations and the confirmation of the theoretically predicted possibilities for the mixing and enhancement of the oscillations [7], double-beta-decay experiments can provide information on the nature of the neutrino and about its absolute mass scale [8–10]. This is a unique feature of double-beta decay, which must be consistent with other scale-fixing measurements like the WMAP measurements [5]. As we are going to discuss later on in this work, there turns out to be a gap between the range of mass limits extracted from double-beta-decay studies, 0.4 to 1.3 eV, and those extracted from the other neutrino-related studies, which are on the order of 0.10 to 0.20 eV or even lower. There is a clear discrepancy between both sets of results concerning the observation of neutrinoless double-beta ($0\nu\beta\beta$) decay. This issue has become a hot one due to the recent claim [11] about the positive identification of $0\nu\beta\beta$ decay signals in the decay of ^{76}Ge (see also [12]). In this work, we discuss constraints, set by the oscillation and mass parameters, on the effective neutrino mass relevant for the $0\nu\beta\beta$ decay and compare them with the ones obtained by performing nuclear-structure study.

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2. FORMALISM

2.1. Neutrino Data

To calculate effective neutrino properties, like the effective electron-neutrino mass, $\langle m_\nu \rangle$, one needs to know the neutrino-mixing matrix U and the light-neutrino mass spectrum (m_1, m_2, m_3) [8]. Out of the very rich recently published list of articles dealing with analysis of the SNO results, we have selected two representative ones, namely, (a) the results presented in the paper of Bandyopadhyay, Choubey, Goswami, and Kar (BCGK) [13] and (b) the expression of the mixing matrix in terms of the solar-neutrino data and the zeroth-order approximation of the mixing matrix assuming maximum mixing to perform our calculations. The next step consists of the definition of a neutrino mass spectrum. The relative order between the mass eigenvalues, usually referred to in the literature as *mass hierarchy or hierarchical order of the mass eigenvalues*, cannot be fixed only by the measured squared mass differences. In order to estimate the possible range of m_i, we define the relative scales

$$m_1 = f m_2, \qquad m_2 = g m_3 \tag{1}$$

for the so-called normal hierarchy $(m_1 \approx m_2 < m_3)$ and

$$m_1 = f m_2, \qquad m_3 = g m_1 \tag{2}$$

for the so-called inverse $(m_1 \approx m_2 > m_3)$ and degenerate $(m_3 \approx m_2 \approx m_1)$ hierarchies. To these factors we have added the information related to the scale of the mass eigenvalues, which is determined by the extreme value

$$m_0 = \Omega_\nu / 3, \tag{3}$$

where the value of Ω_ν is taken from the WMAP data. The factors f and g are determined in such a way that the resulting masses $m_i(f, g)$ obey the observed mass differences, hereafter, denoted as Δm^2

Table 1. Calculated effective electron-neutrino masses $\langle m_\nu \rangle_\pm$ (Indicated in the table are the mass spectrum and the adopted mixing matrix. The values are given in eV. The results listed as "extreme" have been obtained by using the extreme upper values of f and g. The adopted values for the mass differences are $\delta m_{12}^2 = 7.1 \times 10^{-5}$ eV^2, $\delta m_{23}^2 = 2.7 \times 10^{-3}$ eV^2, and $m_0 = 0.24$ eV. The mixing matrix $U(a)$ is taken from [13], $U(b)$ is calculated by taking the largest values of the solar and atmospheric mixing angles, and $U(c)$ is the maximum-mixing solution.)

Mass spectrum	$\langle m_\nu \rangle$	$U(a)$	$U(b)$	$U(c)$
Normal ($m_1 = 0$)	$\langle m_\nu \rangle_-$	−0.010	−0.012	−0.019
	$\langle m_\nu \rangle_+$	0.011	0.012	0.019
(extreme)	$\langle m_\nu \rangle_-$	0.105	0.086	-0.769×10^{-4}
	$\langle m_\nu \rangle_+$	0.231	0.231	0.231
Inverse ($m_3 = 0$)	$\langle m_\nu \rangle_-$	0.105	0.087	-0.153×10^{-2}
	$\langle m_\nu \rangle_+$	0.234	0.235	0.235
(extreme)	$\langle m_\nu \rangle_-$	0.108	0.088	-0.749×10^{-4}
	$\langle m_\nu \rangle_+$	0.237	0.237	0.237
Degenerate (extreme)	$\langle m_\nu \rangle_-$	0.107	0.088	-0.715×10^{-4}
	$\langle m_\nu \rangle_+$	0.237	0.237	0.237

($\Delta m_{31}^2 \approx \Delta m_{32}^2$) and δm^2 (Δm_{12}^2). For each set of allowed values of f, g and for each of the hierarchies considered we have calculated m_i. The effective neutrino mass $\langle m_\nu \rangle$, relevant for the $0\nu\beta\beta$ decay, is given by [14]

$$\langle m_\nu \rangle_\pm = \sum_{i=1}^{3} m_i \lambda_i |U_{ei}|^2 = m_1 U_{e1}^2 \pm m_2 U_{e2}^2, \quad (4)$$

since for the adopted best fit $U_{e3} \approx 0$ [13]. We have consistently neglected CP-violating phases, assumed CP conservation, and written $\lambda_i = \pm 1$ for the relative Majorana phases, since the fit of [13] was performed under the assumption of CP conservation. In Table 1 we give, for each of the adopted forms of the mixing matrix U, the range of values of the calculated effective electron-neutrino masses. These values correspond to the limiting values of f and g given in the previous equations. This part of the analysis is, of course, relevant for the present study since it determines exclusion regions for the allowed values of the effective neutrino mass relevant for the $0\nu\beta\beta$ (see, for instance, [6] for a similar approach).

2.2. Nuclear Matrix Elements

The implication of these results for $\langle m_\nu \rangle$ upon the rates of $0\nu\beta\beta$ decay is easily seen if one writes the corresponding half-life $t_{1/2}^{(0\nu)}$ as

$$\left(t_{1/2}^{(0\nu)}\right)^{-1} = \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2 C_{mm}^{(0\nu)}, \quad (5)$$

where the factor $C_{mm}^{(0\nu)}$ is defined as

$$C_{mm}^{(0\nu)} = G_1^{(0\nu)} \left(M_{\rm GT}^{(0\nu)}(1 - \chi_{\rm F})\right)^2 \quad (6)$$

in terms of the nuclear matrix elements, $M_{\rm GT}^{(0\nu)}(1 - \chi_{\rm F})$, and the phase-space factors $G_1^{(0\nu)}$ entering the mass term of the transition probability [14]. The standard procedure, applied in the literature to calculate the $0\nu\beta\beta$-decay rate, involves three major components:

(a) The calculation of the spectrum of the intermediate double-odd-mass nucleus with $(A, N \pm 1, Z \mp 1)$ nucleons. The pnQRPA is an approximate diagonalization in the one-particle–one-hole $1p$–$1h$ (or two-quasiparticle) space, and it includes the effects of $2p$–$2h$ ground-state correlations by means of the backward-going amplitudes. Since the calculations are based on a quasiparticle mean field, one forces the breaking of certain symmetries like the particle-number symmetry by the use of the BCS approximation and the isospin symmetry by the use of effective proton and neutron single-particle states. The final results of the pnQRPA calculations will certainly be affected by these symmetry-breaking effects induced by the way in which we handle the nuclear interactions [14].

(b) The calculation of the leptonic phase-space factors, as dictated by the second-order perturbative treatment of the electroweak interaction. At the level of the minimal extension of the Standard Model (SM) Lagrangian (mass sector only), these phase-space factors can be easily calculated. At the level of the

Table 2. Calculated nuclear matrix elements for the case of ^{76}Ge (The adopted value for the lower limit of the half-life is $t_{1/2}^{(0\nu)} = 2.5 \times 10^{25}$ yr. Indicated in the table are the models used to calculate the nuclear matrix elements, which are taken from the literature. The abbreviations stand for the proton–neutron quasiparticle random-phase approximation (pnQRPA), the particle-number-projected pnQRPA (pnQRPA (proj.)), proton–neutron pairing pnQRPA (pnQRPA + pn pairing), the renormalized pn-QRPA (RQRPA), the second pnQRPA (SQRPA), the self-consistent renormalized pnQRPA (SCRQRPA), the fully renormalized pnQRPA (full-RQRPA), and the variation after the projection mean-field approach (VAMPIR).)

$C_{mm}^{(0)}$, yr^{-1}	$F_N(\min) \times$ $\times 10^{-12}$	Theory
1.12×10^{-13}	2.80	pnQRPA
6.97×10^{-14}	1.74	pnQRPA
7.51×10^{-14}	1.88	pnQRPA (proj.)
7.33×10^{-14}	1.83	pnQRPA
1.42×10^{-14}	0.35	pnQRPA + + pn pairing
1.18×10^{-13}	2.95	pnQRPA
8.27×10^{-14}	2.07	pnQRPA
2.11×10^{-13}	5.27	RQRPA
6.19×10^{-14}	1.55	RQRPA + q-dep. operators
$(1.8–2.2) \times 10^{-14}$	0.45–0.55	pnQRPA
$(5.5–6.3) \times 10^{-14}$	1.37–1.57	RQRPA
$(2.7–3.2) \times 10^{-15}$	0.07–0.08	SCRQRPA
1.85×10^{-14}	0.46	pnQRPA
1.21×10^{-14}	0.30	RQRPA
3.63×10^{-14}	0.91	full-RQRPA
6.50×10^{-14}	1.62	SQRPA
2.88×10^{-13}	7.20	VAMPIR
1.58×10^{-13}	3.95	Shell Model
1.90×10^{-14}	0.47	Shell Model

two-nucleon mechanism, the value of g_A is currently fixed at $g_A = 1.254$, but, for the medium-heavy and heavy nuclei, an effective value of $g_A = 1.0$ has also been used. In this work we adopt the conservative estimate of $g_A = 1.254$.

(c) The calculation of the matrix elements of the relevant current operators that act upon the nucleons. These operators are also well known and their multipole structures are derived from the expansion of the electroweak current [14]. In the present calculation we have considered the standard type of operators, without introducing momentum dependence.

A compilation of the values of nuclear matrix elements and phase-space factors can be found in [14]. If one compares the extracted upper limits for the neutrino masses from $0\nu\beta\beta$-decay data with the ranges of neutrino masses given in the previous section, it becomes evident that the present generation of $0\nu\beta\beta$ experiments is rather insensitive to the effective neutrino mass coming from the best fit of the solar + atmospheric + reactor data, except for the Heidelberg–Moscow experiment if one takes the range of values ($\langle m_\nu \rangle = 0.11–0.56$ eV) reported in [11]. If one takes the value $\langle m_\nu \rangle \approx 0.24$ eV (the heaviest possible effective mass), which is favored by the inverse and degenerate mass spectra (see Table 1), one sees that it is outside the range of the present upper limits fixed by double-beta-decay experiments, with the possible exception of the decay of ^{76}Ge, which just barely reaches this estimate. To reach the neutrino-mass value resulting from the neutrino data, one definitely needs larger matrix elements than the ones produced thus far by the spherical pnQRPA model and/or longer half-lives than the present measured limits.

2.3. pnQRPA Matrix Elements for ^{76}Ge

Table 2 shows the results of the matrix elements corresponding to the mass sector of the $0\nu\beta\beta$ decay in ^{76}Ge calculated within the family of the pnQRPA-related models [14]. By using the phase-space factors listed in [14], we arrive at the central value for the matrix elements in the pnQRPA, namely,

$$M_{\rm GT}^{(0\nu)}(1-\chi_{\rm F})_{\rm pnQRPA} = 3.65. \quad (7)$$

The corresponding value for the latest large-scale shell-model calculation is given by

$$M_{\rm GT}^{(0\nu)}(1-\chi_{\rm F})_{\rm shell\ model} = 1.74. \quad (8)$$

In terms of the effective neutrino mass, using the half-life $t_{1/2}^{(0\nu)} \geq 2.5 \times 10^{25}$ yr, these matrix elements lead to

$$\langle m_\nu \rangle_{\rm pnQRPA} \leq 0.35 \ \ {\rm eV} \quad (9)$$

for the pnQRPA estimate and

$$\langle m_\nu \rangle_{\rm shell\ model} \leq 0.74 \ \ {\rm eV} \quad (10)$$

for the shell-model estimate of the matrix element. This means that to go to masses of the order of 0.24 eV, as required by WMAP, one needs larger nuclear matrix elements than the ones given by the pnQRPA or by the available shell-model results. In fact, to reach the WMAP limit one would need the value

$$M_{\rm GT}^{(0\nu)}(1-\chi_{\rm F})_{\rm exp} \geq 5.36, \tag{11}$$

which is $\approx \sqrt{2}$ times larger than the reference pnQRPA. The largest matrix element listed in Table 2, coming from the VAMPIR approach, would yield the value $\langle m_\nu \rangle_{\rm VAMPIR} \leq 0.19$ eV, which just touches the value $\langle m_\nu \rangle \leq 0.24$ eV coming from the analysis of the neutrino-related data. However, it is appropriate to point out here that the VAMPIR matrix element is considered unrealistically large, because in the calculations no proton–neutron residual interaction was included.

Finally, our present value

$$M_{\rm GT}^{(0\nu)}(1-\chi_{\rm F})_{\rm pnQRPA}^{\rm present} = 3.33 \tag{12}$$

is consistent with the central value (7), and it yields an effective neutrino mass

$$\langle m_\nu \rangle_{\rm pnQRPA}^{\rm present} \leq 0.39 \ \ {\rm eV} \tag{13}$$

if one takes for the half-life the lower limit and

$$\langle m_\nu \rangle_{\rm pnQRPA}^{\rm present} \leq 0.50 \ \ {\rm eV} \tag{14}$$

if one takes for the half-life the value 1.5×10^{25} yr given by the Heidelberg–Moscow Collaboration [11].

Thus, the issue about the observability of the $0\nu\beta\beta$ decay relies, from the theoretical side, upon the estimates for the effective neutrino mass and upon the estimates of the relevant nuclear matrix elements. While in some cases the differences between the calculated matrix elements are within factors of the order of 3, in some other cases the differences are much larger. This shows one of the essential features of nuclear double-beta decay, namely, that case-by-case theoretical studies are needed instead of a global one [14]. The elucidation of this problem relies on data that may be available in the next generation of double-beta-decay experiments. These future experiments are needed to reach the values of effective neutrino masses extracted from the neutrino-oscillation-related data.

3. CONCLUSIONS

To conclude, in this work we have presented results on the effective neutrino mass, as obtained from the best-fit mass-mixing matrix U determined from the analysis of solar + atmospheric + reactor + satellite data and compared them with the values extracted from neutrinoless double-beta-decay experiments. The value of the effective electron-neutrino mass extracted from the neutrino-related experiments, $\langle m_\nu \rangle \leq 0.24$ eV, does not compare with the central value of $\langle m_\nu \rangle \approx 0.39$ eV, reported in [11]. It does not compare, either, with the values given by the standard pnQRPA model after taking into account the span in the calculated matrix elements. To explain the difference between the above results, we have compiled systematics of the calculated nuclear matrix elements and performed additional pnQRPA calculations. In the case of ^{76}Ge and if one adopts for the half-life the lower limit of 2.5×10^{25} yr, the nuclear matrix elements needed to yield the desired effective neutrino masses are larger than any of the known nuclear matrix elements calculated in the framework of the spherical pnQRPA. This conclusion also holds for the available shell-model results.

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